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# Some Basic Properties of Uniformly Symmetrically Continuous (Real Valued) Functions on Metric Spaces

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**Abstract**. We give the definition of uniform symmetric continuity for real valued functions defined on a metric space. Then we investigate the basic properties of uniformly symmetrically continuous functions and compare them with those of symmetrically continuous functions and uniformly continuous functions. Several examples are also given.

## 1. Introduction

Brown and Page in [1] have defined a metric space. Given a set $X \neq \emptyset$. The function $d: X \times X \to \mathbb{R}$ is called by metric on $X$ if and only if for every $x, y, z \in X$ satisfy:

(i) $d(x, y) \geq 0$, and $d(x, y) = 0$ if and only if $x = y$.
(ii) $d(x, y) = d(y, x)$
(iii) $d(x, y) \leq d(x, z) + d(z, y)$.

The set $X$ which has a metric $d$ on $X$ is called by a metric space, and it is denoted by $(X, d)$. The metric space $(X, d)$ is said to be discrete if $d(x, y) = 1$ for $x \neq y$ and $d(x, y) = 0$ for $x = y$.

The concept of continuity and any concept involving continuity plays a very important role in real analysis, functional analysis, and topology. There are various types of continuities, four of which will be discussed in this paper.

**Definition 1.** *Given metric spaces* $(X, d)$ *and* $(\mathbb{R}, d^*)$, $A \subset X$. *The function* $f: A \to \mathbb{R}$ *is said to be **continuous** at a point* $c \in A$ *if for every* $\varepsilon > 0$ *there exists a* $\delta > 0$ *such that* $d^*(f(x), f(c)) < \varepsilon$, *whenever* $x \in A$, $d(x, c) < \delta$.
*We say that* $f$ *is continuous on* $A$ *if* $f$ *is continuous at each point* $c \in A$.

**Definition 2.** *Given metric spaces* $(X, d)$ *and* $(\mathbb{R}, d^*)$, $A \subset X$. *The function* $f: A \to \mathbb{R}$ *is said to be **uniformly continuous** on* $A$ *if for every* $\varepsilon > 0$ *there exists* $\delta > 0$ *such that* $d^*(f(x), f(y)) < \varepsilon$, *where* $x, y \in A$, *and* $d(x, y) < \delta$.

**Definition 3.** *Given metric spaces* $(X, d)$ *and* $(\mathbb{R}, d^*)$, $A \subset X$. *The function* $f: A \to \mathbb{R}$ *is said to be* ***symmetrically continuous*** *at* $c \in A$ *if for every* $\varepsilon > 0$ *there exists* $\delta > 0$ *such that for all* $h \in \mathbb{R}, |h| < \delta$ *and for every* $x, y \in A$, $d(x,c) = d(y,c) = |h|$ *then*

$$d^*\big(f(x), f(y)\big) < \varepsilon.$$

*We say* $f$ *is symmetrically continuous on* $A$ *if* $f$ *is simmetrically continuous at every point* $c \in A$.

Like continuous functions, functions have been studied thoroughly by many authors, see for example in [1-5,7-13]. In particular in [6], Marcus defined a concept of uniformly symmetrically continuous function on $\mathbb{R}$.

In this paper, we would like to declare the generalization of the concept of uniformly symmetrically continuous, that is the uniformly symmetrically continuous (real valued) function, so that aplicable on metric spaces. The purpose of this study is to obtain properties of uniformly symmetrically continuous function on metric spaces and compare them with those of symmetrically continuous functions and uniformly continuous functions.

## 2. The Result

**Definition 4.** *Given two metric spaces* $(X, d)$ *and* $(\mathbb{R}, d^*)$, $A \subset X$. *The function* $f: A \to \mathbb{R}$ *is said to be uniformly symmetrically continuous on* $A$ *if for every* $\varepsilon > 0$ *there exists* $\delta > 0$ *such that for all* $h \in \mathbb{R}, |h| < \delta$ *and for every* $x, y, z \in A$, $d(x,z) = d(y,z) = |h|$ *then*

$$d^*\big(f(x), f(y)\big) < \varepsilon.$$

The next propositions are the properties of the concept of continuity. First, the relation between the concept of continuity and uniformly continuity is explained

**Proposition 5.** *Given two metric spaces* $(X, d)$ *and* $(\mathbb{R}, d^*)$, $A \subset X$. *If* $f$ *is uniformly continuous on* $A$, *then* $f$ *is continuous on* $A$, *but the convers is not satisfied.*

***Proof:***

This follows immediately from the definition. The convers is not satisfied because there exists the counter example. For example, given the usual metric space $\mathbb{R}$, that is $\mathbb{R}$ with $d(x,y) = |x - y|$. The function $f: \mathbb{R} \to \mathbb{R}$ and

$$f(x) = 1/x$$

is a continuous on interval $(0,1)$, but $f$ is not uniformly continuous on its interval. ■

Second, the relation between the concept of continuity and symmetrically continuity is described as follows

**Proposition 6.** *Given two metric spaces* $(X, d)$ *and* $(\mathbb{R}, d^*)$, $A \subset X$. *If* $f$ *is continuous at* $c \in A$, *then* $f$ *is symmetrically continuous at* $c \in A$, *but the convers is not satisfied..*

***Proof:***

Let $f$ is continuous at $c \in A$. Let $\varepsilon > 0$ be given, there exists $a\ \delta > 0$ such that $d^*\big(f(x), f(c)\big) < \frac{\varepsilon}{3}$ whenever $x \in A$ and $d(x,c) < \delta$. Let $h \in \mathbb{R}$, $|h| < \delta$ and $x, y \in A$, $d(x,c) = d(y,c) = |h|$, we obtain

$$d^*\big(f(x), f(y)\big) \le d^*\big(f(x), f(c)\big) + d^*\big(f(c), f(y)\big) = \frac{\varepsilon}{3} + \frac{\varepsilon}{3} < \varepsilon.$$

It show that $f$ is symmetrically continuous at $c \in A$.

The convers is not satisfied because there exists the counter example. For example (look at [6]), given the usual metric space $\mathbb{R}$, that is $\mathbb{R}$ with $d(x,y) = |x - y|$. The function $f: \mathbb{R} \to \mathbb{R}$ with

$$f(x) = \begin{cases} 0, & x \neq 3 \\ 2, & x = 3 \end{cases}$$

ss symmetrically continuous at $c = 3$, but $f$ is not continuous at $c = 3$. ■

**Corolarry 7.** *Given two metric spaces* $(X,d)$ *and* $(\mathbb{R},d^*)$, $A\subset X$. *If* $f$ *is continuous on* $A$, *then* $f$ *is symmetrically continuous on* $A$.

It is obvious that every uniformly symmetrically continuous function is symmetrically continuous. Let us record this as a proposition.

**Proposition 8.** *Given two metric spaces* $(X,d)$ *and* $(\mathbb{R},d^*)$, $A\subset X$. *If* $f$ *is uniformly symmetrically continuous on* $A$, *then* $f$ *is symmetrically continuous* $A$, *but the convers is not satisfied..*

***Proof:***
This follows immediately from the definition. ■

The next, we want to explain the relation between the concept of uniformly continuity and uniformly symmetrically continuity.

**Proposition 9.** *Given two metric spaces* $(X,d)$ *and* $(\mathbb{R},d^*)$, $A\subset X$. *If* $f$ *is uniformly continuous on* $A$, *then* $f$ *is uniformly symmetrically continuous on* $A$, *but the convers is not satisfied.*

***Proof:***
Let $f$ is uniformly continuous on $A$. Let $\varepsilon>0$ be given, there exists a $\delta>0$ such that $d^*\big(f(x),f(y)\big)<\frac{\varepsilon}{3}$ whenever $x,y\in A$ and $d(x,y)<\delta$. Let $h\in\mathbb{R}$, $|h|<\delta$ and $x,y,z\in A$, $d(x,z)=d(y,z)=|h|$, then we get

$$d^*\big(f(x),f(y)\big)\le d^*\big(f(x),f(z)\big)+d^*\big(f(z),f(y)\big)=\frac{\varepsilon}{3}+\frac{\varepsilon}{3}<\varepsilon.$$

It show that $f$ is uniformly symmetrically continuous on $A$. ■

**Corolarry 10.** *Given two metric spaces* $(X,d)$ *and* $(\mathbb{R},d^*)$, $A\subset X$. *If* $f$ *is uniformly continuous on* $A$, *then* $f$ *is symmerically continuous on* $A$, *but the convers is not satisfied..*

In connection with four types of the continuities mentioned above, the problem arises to investigate their relation. For a given two metric spaces $(X,d)$ and $(\mathbb{R},d^*)$, let $A\subset X$ and we define

$$USC=\{f:A\to\mathbb{R}\mid f \text{ is uniformly symmetrically continuous on } A\}$$
$$UC=\{f:A\to\mathbb{R}\mid f \text{ is uniformly continuous on } A\}$$
$$C=\{f:A\to\mathbb{R}\mid f \text{ is continuous on } A\},$$
$$SC=\{f:A\to\mathbb{R}\mid f \text{ is symmetrically continuous on } A\}.$$

Then we have the following result

$$\begin{array}{rl} UC\subset C & \text{but } \; C\not\subset UC\\ C\subset SC & \text{but } \; SC\not\subset C\\ USC\subset SC & \text{but } \; SC\not\subset USC\\ UC\subset USC & \text{but } \; USC\not\subset UC\end{array}$$

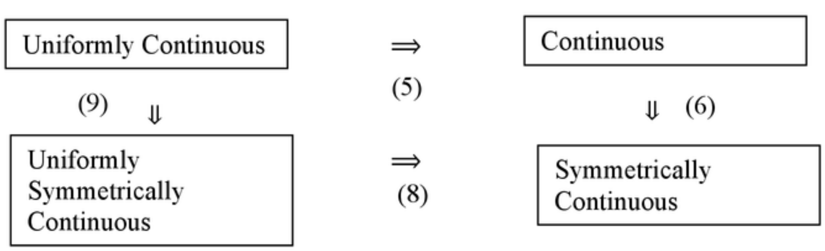


**Figure 1.** The connection between four types of the continuities

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## 3. The Basic Properties

Although uniformly symmetrically continuous functions and uniformly continuous functions are different, they share some basic properties as will be shown in this section.

**Theorem 11:** Given two metric spaces $(X,d)$ and $(\mathbb{R},d*)$; $A\subset X$, The functions $f,g:A \longrightarrow \mathbb{R}$. If $f$ and $g$ is uniformly continuous on $A$ and $d*$ is usual metric $(d(x,y) = |x-y|)$, Then
(a) $f+g$ is uniformly symmetrically continuous on $A$.
(b) $kf$ is uniformly symmetrically continuous on $A$ whenever $k \in \mathbb{R}$.

Theorem 11 above can be proved by Definition 4.

## 4. Conclusion

By the given two metric spaces $(X,d)$ and $(\mathbb{R},d*)$ with $A\subset X$, we have the following conclusion: (1) We success in defining the concept of uniformly symmetrically continuous of the real valued function on a metric space. The concept is an abstraction to domain of the concept of uniformly symmetrically continuous of the real valued function on $\mathbb{R}$, (2) The concept of uniformly symmetrically continuity of the real valued function on a metric space is a generalization of the concept of symmetrically continuity of the real valued function on a metric.

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